

Time-dependent Ginzburg-Landau Equation in the Nambu–Jona-Lasinio Model

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Abstract

We apply the closed time-path Green function formalism in the Nambu–Jona-Lasinio model. First of all, we use this formalism to obtain the well-known gap equation for the quark condensate in a stationary homogeneous system. We have also used this formalism to obtain the Ginzburg-Landau (GL) equation and the time-dependent Ginzburg-Landau (TDGL) equation for the chiral order parameter in an inhomogeneous system. In our derived GL and TDGL equations, there is no other parameters except for those in the original NJL model.

Keywords:

QCD Phase Transitions, Closed Time-path Green Function, Nambu–Jona-Lasinio model, Time-dependent Ginzburg-Landau Equation

1. Introduction

QCD thermodynamics, for example the equation of state of quark gluon plasma (QGP), phase transition of chiral symmetry restoration, deconfinement phase transition and so on, has been a subject of intensive investigations in recent years. On the one hand, the deconfined QGP are expected to be formed in ultrarelativistic heavy-ion collisions [1, 2, 3, 4, 5, 6, 7, 8] (for example the experiments at the Relativistic Heavy Ion Collider (RHIC) and at the Large Hadron Collider (LHC)) and in the interior of neutron stars [9, 10, 11, 12]; On the other hand, studying the thermodynamical behaviors of the QGP, especially the deconfinement and chiral phase transitions, is an elementary problem in strong interaction physics.

Lattice QCD calculations [13, 14, 15, 16] as well as models studies [17, 18, 19, 20, 21, 22, 23, 24, 25] indicate that there is a critical point in the QCD phase diagram in the plane of temperature and baryon chemical potential [26], which separates the first order phase transition at high baryon chemical potential from the continuous crossover at high temperature. Furthermore, experiments with the goal to search for the QCD critical point are planned and underway at RHIC and at the Super Proton Synchrotron (SPS) [27, 28, 29, 30]. Therefore, the critical dynamics of the chiral phase transition has attracted lots of attentions in recent years. Based on equilibrium thermodynamics [31] or quasi-stationary framework [32], signatures of the QCD critical point have been studied by M. A. Stephanov *et al.*. A. Singh *et al.* have also studied the kinetics of the chiral phase transition subsequent to a quench from a disordered phase to the ordered phase [33].

In this work, we will adopt the closed time-path Green function (CTPGF) formalism to derive the Ginzburg-Landau equation and the time-dependent Ginzburg-Landau equation of the order parameter for the chiral phase transition in an inhomogeneous quark matter system in the Nambu–Jona-Lasinio (NJL) model. The CTPGF formalism, developed by Schwinger [34] and Keldysh [35], has been used to solve lots of interesting problems in statistical physics and condensed matter theory [36], and it has also been used in the NJL model to derive the transport equations [37]. It is generally believed that this technique is quite effective in investigating the nonequilibrium statistical theory [36, 38]. It has also been used to treat a system of self-interacting bosons described by $\lambda\phi^4$ scalar fields [39].

The paper is organized as follows. In Sec. 2 we simply review the CTPGF formalism, mainly on the generating functional of the CTPGFs. In Sec. 3 we apply the CTPGF formalism into the NJL model. First of all, we use this formalism to derive the well-known gap equation of the quark condensate for the homogeneous system, then we also obtain the Ginzburg-Landau equation and time-dependent Ginzburg-Landau equation of the order parameter for an inhomogeneous system. In Sec. 4 we present our summary and conclusions.

2. Simple Review on the CTPGF

2.1. Two-point CTPGFs

In this section we give a short review about the CTPGF formalism and introduce some notations which we will use in our following discussions. Readers who are not familiar with the CTPGF formalism are strongly suggested

to reference the excellent review article by K. C. Chou *et al.* [36]. Here, we take the real boson field $\varphi(x)$ for example. The two-point CTPGF is defined as

$$\begin{aligned} G(x, y) &\equiv -i\text{Tr}\{T_p(\varphi(x)\varphi(y))\hat{\rho}\} \\ &\equiv -i\langle T_p(\varphi(x)\varphi(y)) \rangle, \end{aligned} \quad (1)$$

where $\hat{\rho}$ is the density matrix and T_p is the time ordering operator along the closed time-path p which goes from $-\infty$ to $+\infty$ (also denoted as the positive time branch) and then returns back from $+\infty$ to $-\infty$ (negative time branch). We should note that any event at the negative time branch is later than any that at the positive time branch. The Green function $G(x, y)$ in Eq.(1) on the closed time-path p can be expressed as Green function whose coordinates x and y are constrained to be on either positive or negative time branches, and then $G(x, y)$ becomes a 2×2 matrix, i.e.,

$$\hat{G}(x, y) \equiv \begin{pmatrix} G_{++} & G_{+-} \\ G_{-+} & G_{--} \end{pmatrix} \equiv \begin{pmatrix} G_F & G_+ \\ G_- & G_{\tilde{F}} \end{pmatrix} \quad (2)$$

with

$$G_F(x, y) \equiv -i\langle T(\varphi(x)\varphi(y)) \rangle, \quad (3)$$

$$G_+(x, y) \equiv -i\langle \varphi(y)\varphi(x) \rangle, \quad (4)$$

$$G_-(x, y) \equiv -i\langle \varphi(x)\varphi(y) \rangle, \quad (5)$$

$$G_{\tilde{F}}(x, y) \equiv -i\langle \tilde{T}(\varphi(x)\varphi(y)) \rangle, \quad (6)$$

where T is the usual time-ordering operator and \tilde{T} is anti-time-ordering operator. Therefore, G_F is the conventional Feynman causal propagator and $G_{\tilde{F}}$ is an anti-causal propagator, whose expression can be given explicitly as

$$G_{\tilde{F}}(x, y) = -i\theta(y_0, x_0)\langle \varphi(x)\varphi(y) \rangle - i\theta(x_0, y_0)\langle \varphi(y)\varphi(x) \rangle. \quad (7)$$

Furthermore, It can be easily proved that we have the following identity, i.e.,

$$G_F(x, y) + G_{\tilde{F}}(x, y) = G_+(x, y) + G_-(x, y). \quad (8)$$

So far we have presented CTPGFs in two representations: one is the closed time-path representation, i.e., Eq.(1) and the other is the single time representation given by Eq.(2). In fact, there is another representation which is

directly related with measurable quantities, which is often called as physical representation and defined as

$$G_r(x, y) \equiv -i\theta(x_0, y_0)\langle[\varphi(x), \varphi(y)]\rangle, \quad (9)$$

$$G_a(x, y) \equiv i\theta(y_0, x_0)\langle[\varphi(x), \varphi(y)]\rangle, \quad (10)$$

$$G_c(x, y) \equiv -i\langle\{\varphi(x), \varphi(y)\}\rangle. \quad (11)$$

G_r , G_a , and G_c are retarded, advanced, and correlation Green functions, respectively. The relations between the CTPGFs in the physical representation and those in the single time representation are given by

$$G_r = G_F - G_+ = G_- - G_{\tilde{F}}, \quad (12)$$

$$G_a = G_F - G_- = G_+ - G_{\tilde{F}}, \quad (13)$$

$$G_c = G_F + G_{\tilde{F}} = G_+ + G_-, \quad (14)$$

and the inverse relations are

$$\hat{G} = \frac{1}{2}G_r \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} + \frac{1}{2}G_a \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} + \frac{1}{2}G_c \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}. \quad (15)$$

Introducing two-component vectors

$$\xi \equiv \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \eta \equiv \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \quad (16)$$

ones can express Eq.(15) as

$$G_{\alpha\beta} = \frac{1}{2}G_r\xi_\alpha\eta_\beta + \frac{1}{2}G_a\eta_\alpha\xi_\beta + \frac{1}{2}G_c\xi_\alpha\xi_\beta, \quad (17)$$

where $G_{\alpha\beta}$ with Greek subscripts $\alpha, \beta = \pm$ are components of the CTPGF matrix in single time representation in Eq.(2).

2.2. Generating functionals of CTPGFs

We begin with the Lagrangian density of the real boson field as follows

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}\partial_\mu\varphi(x)\partial^\mu\varphi(x) - \frac{1}{2}m^2\varphi^2(x) + \mathcal{L}_{int}(\varphi(x)) \\ &= \mathcal{L}_0(\varphi(x)) + \mathcal{L}_{int}(\varphi(x)), \end{aligned} \quad (18)$$

where \mathcal{L}_0 is the free field term and \mathcal{L}_{int} is the interaction term. The generating functional for the CTPGFs is defined as

$$Z[J(x)] \equiv \text{Tr} \left\{ T_p \left[\exp \left(i \int_p d^4 x J(x) \varphi(x) \right) \right] \hat{\rho} \right\}, \quad (19)$$

where the time integration is

$$\begin{aligned} \int_p dt &= \int_{-\infty}^{+\infty} dt_+ + \int_{+\infty}^{-\infty} dt_- \\ &= \int_{-\infty}^{+\infty} dt_+ - \int_{-\infty}^{+\infty} dt_-. \end{aligned} \quad (20)$$

The n -point CTPGF is defined as

$$\begin{aligned} G_p(1 \cdots n) &\equiv (-i)^{n-1} \text{Tr} \left[T_p(\varphi(1) \cdots \varphi(n)) \hat{\rho} \right] \\ &= i(-1)^n \frac{\delta^n Z[J(x)]}{\delta J(1) \cdots \delta J(n)} \Big|_{J=0}. \end{aligned} \quad (21)$$

In the interaction picture the generating functional $Z[J(x)]$ in Eq.(19) can be expressed as

$$\begin{aligned} Z[J(x)] &= \text{Tr} \left\{ T_p \left[\exp \left(i \int_p d^4 x (\mathcal{L}_{int}(\varphi_I(x)) + J(x) \varphi_I(x)) \right) \right] \hat{\rho} \right\} \\ &= \exp \left[i \int_p d^4 x \mathcal{L}_{int} \left(-i \frac{\delta}{\delta J(x)} \right) \right] \\ &\times \text{Tr} \left\{ T_p \left[\exp \left(i \int_p d^4 x J(x) \varphi_I(x) \right) \right] \hat{\rho} \right\}. \end{aligned} \quad (22)$$

Using the Generalized Wick theorem [40], one can show [41, 42, 36]

$$T_p \left[\exp \left(i \int_p d^4 x J(x) \varphi_I(x) \right) \right] = Z_0[J(x)] : \exp \left[i \int_p d^4 x J(x) \varphi_I(x) \right] :, \quad (23)$$

with

$$Z_0[J(x)] = \int_p [d\varphi(x)] \exp \left[i \int_p d^4 x (\mathcal{L}_0(\varphi(x)) + J(x) \varphi(x)) \right]. \quad (24)$$

Here $::$ denotes the normal product. Substituting Eq.(23) into Eq.(22), one obtains

$$Z[J(x)] = \exp \left[i \int_p d^4x \mathcal{L}_{int} \left(-i \frac{\delta}{\delta J(x)} \right) \right] Z_0[J(x)] N[J(x)], \quad (25)$$

here

$$\begin{aligned} N[J(x)] &= \text{Tr} \left\{ : \exp \left[i \int_p d^4x J(x) \varphi_I(x) \right] : \hat{\rho} \right\} \\ &\equiv \exp(iW_p^N[J(x)]) \end{aligned} \quad (26)$$

with

$$W_p^N[J(x)] = \sum_{n=1}^{\infty} \frac{i^{n-1}}{n!} \int_p d1 \cdots \int_p dn J(1) \cdots J(n) \text{Tr}[: \varphi_I(1) \cdots \varphi_I(n): \hat{\rho}]_c, \quad (27)$$

where $N[J(x)]$ is the correlation functional for the initial state.

After some calculations, the generating functional $Z[J(x)]$ in Eq.(25) can be expressed as [42]

$$\begin{aligned} Z[J(x)] &= \int_p [d\varphi(x)] \exp \left[i \int_p d^4x \left(\mathcal{L}_0(\varphi(x)) + \mathcal{L}_{int}(\varphi(x)) + J(x)\varphi(x) \right) \right] \\ &\times \exp \left[iW_p^N \left(- \int_p d^4y G_0^{-1}(x, y) \varphi(y) \right) \right], \end{aligned} \quad (28)$$

where G_0^{-1} is the inverse of the propagator of the real boson field, given by

$$G_0^{-1}(x, y) = (-\partial_\mu \partial^\mu - m^2) \delta_p(x - y). \quad (29)$$

In the single time presentation $\delta_p(x - y)$ can be expressed as

$$\delta_p(x - y) = \delta(x - y) \sigma_3. \quad (30)$$

The generating functional for the connected CTPGFs is given by

$$W[J(x)] = -i \ln Z[J(x)]. \quad (31)$$

Correspondingly, one can obtain

$$\begin{aligned} G_p^c(1 \cdots n) &\equiv (-i)^{n-1} \text{Tr} \left[T_p(\varphi(1) \cdots \varphi(n)) \hat{\rho} \right]_c \\ &= (-1)^{n-1} \frac{\delta^n W[J(x)]}{\delta J(1) \cdots \delta J(n)} \Big|_{J=0}. \end{aligned} \quad (32)$$

For $n = 1$ we have

$$\langle \varphi(x) \rangle = \frac{\delta W[J(x)]}{\delta J(x)}. \quad (33)$$

The vertex generating functional can be obtained by performing the Legendre transformation as follows

$$\Gamma[\langle \varphi(x) \rangle] = W[J(x)] - \int_p d^4x J(x) \langle \varphi(x) \rangle. \quad (34)$$

It can be easily found that

$$\frac{\delta \Gamma[\langle \varphi(x) \rangle]}{\delta \langle \varphi(x) \rangle} = -J(x). \quad (35)$$

Next, we express the external source term in the generating functional in Eq.(28) in the single time representation as

$$\begin{aligned} I_s &= \int_p d^4x J(x) \varphi(x) \\ &= \int_{-\infty}^{+\infty} dt d^3x (J_+(x) \varphi_+(x) - J_-(x) \varphi_-(x)) \\ &= \int d^4x \hat{J}^\dagger(x) \sigma_3 \hat{\varphi}(x), \end{aligned} \quad (36)$$

with

$$\hat{\varphi}(x) = \begin{pmatrix} \varphi_+(x) \\ \varphi_-(x) \end{pmatrix}, \quad \hat{J}(x) = \begin{pmatrix} J_+(x) \\ J_-(x) \end{pmatrix}. \quad (37)$$

Introducing

$$\begin{aligned} J_\Delta(x) &\equiv J_+(x) - J_-(x), & J_c(x) &\equiv \frac{1}{2}(J_+(x) + J_-(x)), \\ \varphi_\Delta(x) &\equiv \varphi_+(x) - \varphi_-(x), & \varphi_c(x) &\equiv \frac{1}{2}(\varphi_+(x) + \varphi_-(x)), \end{aligned} \quad (38)$$

one can express I_s as

$$I_s = \int d^4x (J_\Delta(x) \varphi_c(x) + J_c(x) \varphi_\Delta(x)). \quad (39)$$

Then we can obtain the generating functional for the CTPGFs in the physical presentation as follows

$$Z[J_\Delta(x), J_c(x)] = Z[J_+(x), J_-(x)]. \quad (40)$$

In the same way, we have

$$W[J_\Delta(x), J_c(x)] = -i \ln Z[J_\Delta(x), J_c(x)]. \quad (41)$$

and

$$\begin{aligned} \Gamma[\langle\varphi_\Delta(x)\rangle, \langle\varphi_c(x)\rangle] &= W[J_\Delta(x), J_c(x)] \\ &- \int d^4x (J_\Delta(x)\langle\varphi_c(x)\rangle + J_c(x)\langle\varphi_\Delta(x)\rangle), \end{aligned} \quad (42)$$

with

$$\langle\varphi_c(x)\rangle = \frac{\delta W[J_\Delta(x), J_c(x)]}{\delta J_\Delta(x)}, \quad \langle\varphi_\Delta(x)\rangle = \frac{\delta W[J_\Delta(x), J_c(x)]}{\delta J_c(x)}. \quad (43)$$

Therefore, one can also obtain

$$\frac{\delta\Gamma[\langle\varphi_\Delta(x)\rangle, \langle\varphi_c(x)\rangle]}{\delta\langle\varphi_\Delta(x)\rangle} = -J_c(x), \quad \frac{\delta\Gamma[\langle\varphi_\Delta(x)\rangle, \langle\varphi_c(x)\rangle]}{\delta\langle\varphi_c(x)\rangle} = -J_\Delta(x). \quad (44)$$

3. Application of the CTPGFs in the NJL model

In this section we use the CTPGFs to study the spacially and temporally inhomogeneous quark matter in the NJL model. The Lagrangian density for the two-flavor NJL model is given by [43, 44, 45, 46, 47, 48]

$$\mathcal{L}_{NJL} = \bar{\psi}(i\gamma_\mu\partial^\mu - \hat{m}_0)\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right], \quad (45)$$

where $\psi = (\psi_u, \psi_d)^T$ is the quark field, and $\hat{m}_0 = \text{diag}(m_u, m_d)$ is the current quark mass matrix. Throughout this work, we take $m_u = m_d \equiv m_0$, assuming that the isospin symmetry is reserved on the Lagrangian level. The four-fermion interaction with an effective coupling strength G for the scalar and pseudoscalar channels has $\text{SU}_V(2) \times \text{SU}_A(2) \times \text{U}_V(1)$ symmetry, which is broken to $\text{SU}_V(2) \times \text{U}_V(1)$ when $m_0 \neq 0$. Here $\tau^a (a = 1, 2, 3)$ in the Lagrangian density in Eq.(45) are Pauli matrices in flavor space.

Following the CTPGF formalism in Sec. 2, we can obtain the generating functional for the NJL model as follows

$$\begin{aligned} Z[h, h^a; \bar{\eta}, \eta] &= \int_p [d\bar{\psi}][d\psi] \exp \left\{ i \int_p d^4x \left(\mathcal{L}_0(\bar{\psi}, \psi) + \mathcal{L}_{int}(\bar{\psi}, \psi) + \bar{\eta}\psi + \bar{\psi}\eta \right. \right. \\ &\quad \left. \left. + h\bar{\psi}\psi + h^a\bar{\psi}i\gamma_5\tau^a\psi + W_p^N[-\bar{\psi}\overleftarrow{S}_0^{-1}, -\overrightarrow{S}_0^{-1}\psi] \right) \right\}, \end{aligned} \quad (46)$$

with

$$\begin{aligned}\mathcal{L}_0(\bar{\psi}, \psi) &= \bar{\psi}(i\gamma_\mu\partial^\mu - \hat{m}_0)\psi, \\ \mathcal{L}_{int}(\bar{\psi}, \psi) &= G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right],\end{aligned}\quad (47)$$

where $\bar{\eta}$, η are the external sources for field ψ and $\bar{\psi}$, respectively; h , h^a for composite operators $\bar{\psi}\psi$ and $\bar{\psi}i\gamma_5\tau^a\psi$. The term W_p^N in Eq.(46) is due to initial correlations as shown in the last section, and here S_0^{-1} is the inverse of the propagator for the free fermion field. Up to a constant, we have

$$\begin{aligned}& \exp\left\{i\int_p d^4x\left(G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right] + h\bar{\psi}\psi + h^a\bar{\psi}i\gamma_5\tau^a\psi\right)\right\} \\ &= \int_p [d\sigma][d\pi^a] \exp\left\{i\int_p d^4x\left(-G(\sigma^2 + \pi^{a2}) + (2G\bar{\psi}\psi + h)\sigma\right.\right. \\ &+ \left.\left.(2G\bar{\psi}i\gamma_5\tau^a\psi + h^a)\pi^a - \frac{1}{4G}(h^2 + h^{a2})\right)\right\}.\end{aligned}\quad (48)$$

Substituting Eq.(48) into Eq.(46), we obtain

$$\begin{aligned}Z[h, h^a; \bar{\eta}, \eta] &= \exp\left[-\frac{i}{4G}\int_p d^4x(h^2 + h^{a2})\right] \int_p [d\sigma][d\pi^a] \int_p [d\bar{\psi}][d\psi] \\ &\times \exp\left\{i\int_p d^4x\left(\bar{\psi}S^{-1}\psi - G(\sigma^2 + \pi^{a2}) + h\sigma + h^a\pi^a\right.\right. \\ &+ \left.\left.\bar{\eta}\psi + \bar{\psi}\eta + W_p^N[-\bar{\psi}\overleftarrow{S}_0^{-1}, -\overrightarrow{S}_0^{-1}\psi]\right)\right\},\end{aligned}\quad (49)$$

with

$$S^{-1}(x, y) = [i\gamma_\mu\partial^\mu - (\hat{m}_0 - 2G\sigma) + 2Gi\gamma_5\pi^a\tau^a]\delta_p(x - y). \quad (50)$$

In the realistic calculations for the equilibrium (or local equilibrium) system, the contribution of the density matrix, i.e., the term W_p^N in Eq.(49) can be neglected temporarily for simplicity, and its contribution can be recovered through the fluctuation-dissipation theorem satisfied by the CTPGFs. More detailed discussions can be found in Refs. [36, 49]. Neglecting W_p^N in Eq.(49) temporarily, we can integrate over the fermion field and obtain

$$\begin{aligned}Z[h, h^a; \bar{\eta}, \eta] &= \exp\left[-\frac{i}{4G}\int_p d^4x(h^2 + h^{a2})\right] \int_p [d\sigma][d\pi^a] \\ &\times \exp\left\{i\left[I_0(\sigma, \pi^a) + I_{\det}(\sigma, \pi^a)\right.\right. \\ &+ \left.\left.\int_p d^4x(h\sigma + h^a\pi^a) - \int_p d^4xd^4y\bar{\eta}(x)S(x, y)\eta(y)\right]\right\},\end{aligned}\quad (51)$$

with

$$\begin{aligned} I_0(\sigma, \pi^a) &= - \int_p d^4x G(\sigma^2 + \pi^{a2}), \\ I_{\text{det}}(\sigma, \pi^a) &= -i \text{Tr} \ln S^{-1}, \end{aligned} \quad (52)$$

where Tr means trace operation in both the inner space (Dirac, flavor, and color) and the coordinate space. Then we can introduce the effective action for the σ and π fields as follows

$$I^{\text{eff}}(\sigma, \pi^a) = I_0(\sigma, \pi^a) + I_{\text{det}}(\sigma, \pi^a). \quad (53)$$

In the mean field approximation, the vertex generating functional can be approximated by the effective action [50], i.e.,

$$\Gamma(\sigma, \pi^a) \simeq I^{\text{eff}}(\sigma, \pi^a), \quad (54)$$

where σ and π^a mean $\langle \sigma \rangle$ and $\langle \pi^a \rangle$, without confusions. In the physical representation we have

$$\begin{aligned} \Gamma(\sigma_\Delta, \pi_\Delta^a; \sigma_c, \pi_c^a) &\simeq I^{\text{eff}}(\sigma_\Delta, \pi_\Delta^a; \sigma_c, \pi_c^a) \\ &= I_0(\sigma_\Delta, \pi_\Delta^a; \sigma_c, \pi_c^a) + I_{\text{det}}(\sigma_\Delta, \pi_\Delta^a; \sigma_c, \pi_c^a), \end{aligned} \quad (55)$$

with

$$I_0(\sigma_\Delta, \pi_\Delta^a; \sigma_c, \pi_c^a) = -2G \int d^4x (\sigma_\Delta \sigma_c + \pi_\Delta^a \pi_c^a), \quad (56)$$

and

$$\begin{aligned} [S^{-1}(x, y)]_{\alpha\beta} &= \left[(i\gamma_\mu \partial^\mu - \hat{m}_0) \xi_\alpha \eta_\beta + 2G(\xi_\alpha \sigma_c + \frac{1}{2} \eta_\alpha \sigma_\Delta) \eta_\beta \right. \\ &\quad \left. + 2Gi\gamma_5 \tau^a (\xi_\alpha \pi_c^a + \frac{1}{2} \eta_\alpha \pi_\Delta^a) \eta_\beta \right] \delta_{\alpha\beta} \delta^4(x - y). \end{aligned} \quad (57)$$

Following the notations in the last section, we have

$$\begin{aligned} \sigma_\Delta(x) &= \sigma_+(x) - \sigma_-(x), & \sigma_c(x) &= \frac{1}{2}(\sigma_+(x) + \sigma_-(x)), \\ \pi_\Delta^a(x) &= \pi_+^a(x) - \pi_-^a(x), & \pi_c^a(x) &= \frac{1}{2}(\pi_+^a(x) + \pi_-^a(x)). \end{aligned} \quad (58)$$

In the following, we neglect the influence of the π^a field and concentrate on the order parameter of chiral phase transition, i.e., the σ field. First of all,

let us consider the simplest stationary homogeneous quark system, i.e., the σ field is independent of the time and space coordinates. From Eq.(44) and Eq.(54) we have

$$\left. \frac{\delta I^{\text{eff}}(\sigma_\Delta, \sigma_c)}{\delta \sigma_\Delta} \right|_{\sigma_\Delta=0, \sigma_c=\sigma} = -h_c = 0, \quad (59)$$

For the left hand side of Eq.(59), we have

$$\frac{\delta I_0(\sigma_\Delta, \sigma_c)}{\delta \sigma_\Delta} = -2G\sigma_c, \quad (60)$$

and

$$\frac{\delta I_{\text{det}}(\sigma_\Delta, \sigma_c)}{\delta \sigma_\Delta} = -i\text{Tr}\left(S \frac{\delta S^{-1}}{\delta \sigma_\Delta}\right). \quad (61)$$

From Eq.(57), we have

$$\frac{\delta[S^{-1}(y, z)]_{\alpha\beta}}{\delta \sigma_\Delta(x)} = G\delta_{\alpha\beta}\delta^4(y-x)\delta^4(y-z). \quad (62)$$

Then

$$\begin{aligned} \frac{\delta I_{\text{det}}(\sigma_\Delta, \sigma_c)}{\delta \sigma_\Delta(x)} &= -i \int_p d^4y d^4z \text{tr} \left[S(y, z) \frac{\delta S^{-1}(y, z)}{\delta \sigma_\Delta(x)} \right], \\ &= -i \int d^4y d^4z \text{tr} \left[\sigma_3^p S(y, z) \sigma_3^p \frac{\delta S^{-1}(y, z)}{\delta \sigma_\Delta(x)} \right] \\ &= -i \int d^4y d^4z \text{tr} \left[S(y, z) \frac{\delta S^{-1}(y, z)}{\delta \sigma_\Delta(x)} \right], \end{aligned} \quad (63)$$

where tr means trace operation not including in the coordinate space; σ_3^p is the Pauli matrix in the single time representation. Substituting Eq.(17) and Eq.(62) into above equation, we obtain

$$\frac{\delta I_{\text{det}}(\sigma_\Delta, \sigma_c)}{\delta \sigma_\Delta(x)} = -iG\text{tr}S_c(x, x). \quad (64)$$

For the inverse of the quark propagator in Eq.(50) without π^a field, we can easily obtain

$$\begin{aligned} S_r(p) - S_a(p) &= S_-(p) - S_+(p), \\ &= -i\pi \frac{\not{p} + M}{E_p} [\delta(p_0 - E_p) - \delta(p_0 + E_p)], \end{aligned} \quad (65)$$

in the momentum space. Here we have

$$M = m_0 - 2G\sigma, \quad E_p = \sqrt{p^2 + M^2}. \quad (66)$$

Employing the fluctuation-dissipation theorem [36], we obtain

$$\begin{aligned} S_c(p) &= \tanh\left[\frac{\beta}{2}(p_0 - \mu)\right][S_r(p) - S_a(p)], \\ &= -i\pi \frac{\not{p} + M}{E_p} [\delta(p_0 - E_p) - \delta(p_0 + E_p)] \tanh\left[\frac{\beta}{2}(p_0 - \mu)\right], \end{aligned} \quad (67)$$

where $\beta = 1/T$ is the inverse of the temperature, μ the quark chemical potential. Substituting Eq.(67) into Eq.(64), we obtain

$$\begin{aligned} \frac{\delta I_{\text{det}}(\sigma_\Delta, \sigma_c)}{\delta \sigma_\Delta(x)} &= -iG \int \frac{d^4 p}{(2\pi)^4} \text{tr} S_c(p), \\ &= -4GN_f N_c \int \frac{d^3 p}{(2\pi)^3} \frac{M}{E_p} \left[1 - \frac{1}{e^{\beta(E_p - \mu)} + 1} \right. \\ &\quad \left. - \frac{1}{e^{\beta(E_p + \mu)} + 1} \right], \end{aligned} \quad (68)$$

where N_f is the number of quark flavor, N_c the color number. Combining Eqs.(59)(60) with Eq.(68), we obtain

$$\sigma = -2N_f N_c \int \frac{d^3 p}{(2\pi)^3} \frac{M}{E_p} \left[1 - \frac{1}{e^{\beta(E_p - \mu)} + 1} - \frac{1}{e^{\beta(E_p + \mu)} + 1} \right], \quad (69)$$

which is the gap equation for the order parameter in the NJL model.

In the following we will consider the weak non-stationary inhomogeneous quark matter, where the “weak” means that we can expand the left hand side of Eq.(59) as power series of the gradient of the $\sigma(t, \vec{x})$ field. The efficiency of this expansion has been verified in Ref. [49]. Furthermore, we only concentrate on the quark system which is near the chiral phase transition, therefore, the order parameter, i.e., the expectation value of the $\sigma(t, \vec{x})$ field is smaller than the critical temperature. Then, the left hand side of Eq.(59) can be expressed as

$$\begin{aligned} \frac{\delta I^{\text{eff}}(\sigma_\Delta, \sigma_c)}{\delta \sigma_\Delta(x)} \Big|_{\sigma_\Delta=0, \sigma_c=\sigma(x)} &\simeq \frac{\delta I^{\text{eff}}(\sigma_\Delta, \sigma_c)}{\delta \sigma_\Delta(x)} \Big|_{\sigma_\Delta=0, \sigma_c=\sigma} \\ &+ \int d^4 y \frac{\delta^2 I^{\text{eff}}(\sigma_\Delta, \sigma_c)}{\delta \sigma_\Delta(x + \frac{y}{2}) \delta \sigma_c(x - \frac{y}{2})} \left[\sigma(x - \frac{y}{2}) \right. \\ &\quad \left. - \sigma(x + \frac{y}{2}) \right], \end{aligned} \quad (70)$$

where the dependence of the σ field in the fermion propagator on the coordinate is neglected in the first term of the right hand side of Eq.(70), i.e., it is just the result for the homogeneous system with σ replaced by $\sigma(x)$. The second term of the right hand side of Eq.(70) can be expressed as

$$\begin{aligned}
& \int d^4y \frac{\delta^2 I^{\text{eff}}(\sigma_\Delta, \sigma_c)}{\delta\sigma_\Delta(x + \frac{y}{2})\delta\sigma_c(x - \frac{y}{2})} \left[\sigma(x - \frac{y}{2}) - \sigma(x + \frac{y}{2}) \right] \\
& \simeq \int d^4y \frac{\delta^2 I^{\text{eff}}(\sigma_\Delta, \sigma_c)}{\delta\sigma_\Delta(x + \frac{y}{2})\delta\sigma_c(x - \frac{y}{2})} \left[-y^\mu \partial_\mu \sigma(x) + \frac{1}{2} y^\mu y^\nu \partial_\mu \partial_\nu \sigma(x) \right] \\
& = \int d^4y \frac{\delta^2 I^{\text{eff}}(\sigma_\Delta, \sigma_c)}{\delta\sigma_\Delta(x + \frac{y}{2})\delta\sigma_c(x - \frac{y}{2})} \left[- \left(-i \frac{\partial}{\partial q_\mu} e^{iqy} \right)_{q=0} \partial_\mu \sigma(x) \right. \\
& + \left. \frac{1}{2} (-i)^2 \left(\frac{\partial^2}{\partial q_\mu \partial q_\nu} e^{iqy} \right)_{q=0} \partial_\mu \partial_\nu \sigma(x) \right] \\
& = i \left[\frac{\partial}{\partial q_\mu} \Gamma_r(x, q) \right]_{q=0} \partial_\mu \sigma(x) - \frac{1}{2} \left[\frac{\partial^2}{\partial q_\mu \partial q_\nu} \Gamma_r(x, q) \right]_{q=0} \partial_\mu \partial_\nu \sigma(x), \quad (71)
\end{aligned}$$

with

$$\Gamma_r(x, q) = \int d^4y \frac{\delta^2 I^{\text{eff}}(\sigma_\Delta, \sigma_c)}{\delta\sigma_\Delta(x + \frac{y}{2})\delta\sigma_c(x - \frac{y}{2})} e^{iqy}, \quad (72)$$

where $\mu, \nu = 0, 1, 2, 3$. We have

$$\frac{\delta^2 I^{\text{eff}}(\sigma_\Delta, \sigma_c)}{\delta\sigma_\Delta(x + \frac{y}{2})\delta\sigma_c(x - \frac{y}{2})} = \frac{\delta^2 I_0(\sigma_\Delta, \sigma_c)}{\delta\sigma_\Delta(x + \frac{y}{2})\delta\sigma_c(x - \frac{y}{2})} + \frac{\delta^2 I_{\text{det}}(\sigma_\Delta, \sigma_c)}{\delta\sigma_\Delta(x + \frac{y}{2})\delta\sigma_c(x - \frac{y}{2})}, \quad (73)$$

with

$$\frac{\delta^2 I_0(\sigma_\Delta, \sigma_c)}{\delta\sigma_\Delta(x + \frac{y}{2})\delta\sigma_c(x - \frac{y}{2})} = -2G\delta^4(y), \quad (74)$$

and

$$\frac{\delta^2 I_{\text{det}}(\sigma_\Delta, \sigma_c)}{\delta\sigma_\Delta(x + \frac{y}{2})\delta\sigma_c(x - \frac{y}{2})} = i \text{Tr} \left[S \frac{\delta S^{-1}}{\delta\sigma_\Delta(x + \frac{y}{2})} S \frac{\delta S^{-1}}{\delta\sigma_c(x - \frac{y}{2})} \right]. \quad (75)$$

After some calculations, we find

$$\Gamma_r(x, q) = -2G + i2G^2 \int \frac{d^4p}{(2\pi)^4} \text{tr} [S_c(x, p) S_r(x, p + q) + S_a(x, p) S_c(x, p + q)]. \quad (76)$$

The correlation Green function for quark field S_c is given in Eq.(67), and the retarded and the advanced Green functions are given by

$$\begin{aligned} S_r(p) &= \frac{1}{\not{p} - M + i\epsilon} + i\pi \frac{\not{p} + M}{E_p} \delta(p_0 + E_p), \\ S_a(p) &= \frac{1}{\not{p} - M + i\epsilon} + i\pi \frac{\not{p} + M}{E_p} \delta(p_0 - E_p). \end{aligned} \quad (77)$$

Here, the dependence of the Green functions on the space and time coordinates is realized through the $\sigma(x)$ field as shown in Eq.(66). Performing calculations on Eq.(76), we obtain

$$\begin{aligned} \Gamma_r(x, q) &= -2G \\ &+ 4G^2 N_c N_f \int \frac{d^3 p}{(2\pi)^3} \left(\tanh\left[\frac{\beta}{2}(E_p - \mu)\right] + \tanh\left[\frac{\beta}{2}(E_p + \mu)\right] \right) \\ &\times \frac{1}{E_p} \left(\frac{2M^2 + E_p q_0 - \vec{p} \cdot \vec{q}}{(q_0 + E_p)^2 - E_{p+q}^2 + i\epsilon(q_0 + E_p)} \right. \\ &\left. + \frac{2M^2 - E_p q_0 - \vec{p} \cdot \vec{q}}{(q_0 - E_p)^2 - E_{p+q}^2 + i\epsilon(q_0 - E_p)} \right), \end{aligned} \quad (78)$$

When considering the variation with respect to the space coordinates, we can simplify Eq.(78) further by taking the limit $q_0 \rightarrow 0$ firstly, which gives

$$\begin{aligned} \Gamma_r(x, q)_{q_0=0} &= -2G \\ &+ 4G^2 N_c N_f \int \frac{d^3 p}{(2\pi)^3} \left(\tanh\left[\frac{\beta}{2}(E_p - \mu)\right] + \tanh\left[\frac{\beta}{2}(E_p + \mu)\right] \right) \\ &\times \frac{2M^2 - \vec{p} \cdot \vec{q}}{E_p E_{p+q}} \left(\mathcal{P} \frac{1}{E_p - E_{p+q}} - \mathcal{P} \frac{1}{E_p + E_{p+q}} \right), \end{aligned} \quad (79)$$

where \mathcal{P} denotes main value, and

$$E_{p+q} = [(\vec{p} + \vec{q})^2 + M^2]^{1/2}. \quad (80)$$

when $|\vec{q}| \rightarrow 0$, we can expand the left hand side of Eq.(79) as power series of $|\vec{q}|$. Furthermore, taking into consideration that the system is near the chiral phase transition, so the quark mass M is smaller than the critical temperature, we obtain

$$\Gamma_r(x, q)_{q_0=0} \simeq -2G$$

$$\begin{aligned}
& + 4G^2 N_c N_f \frac{1}{\pi^2} \int_0^\Lambda dp \frac{p^2}{E_p} \left[1 - \frac{1}{e^{\beta(E_p - \mu)} + 1} - \frac{1}{e^{\beta(E_p + \mu)} + 1} \right] \\
& - |\vec{q}|^2 G^2 N_c N_f \frac{1}{\pi^2} \int_0^\Lambda dp \frac{1}{E_p} \left[1 - \frac{1}{e^{\beta(E_p - \mu)} + 1} - \frac{1}{e^{\beta(E_p + \mu)} + 1} \right],
\end{aligned} \tag{81}$$

where Λ is the cut-off of the momentum integration which is a parameter in the NJL model. From Eq.(81) we can easily obtain

$$\left[\frac{\partial}{\partial q^i} \Gamma_r(x, q) \right]_{q=0} = 0, \tag{82}$$

$$\begin{aligned}
\left[\frac{\partial^2}{\partial q^i \partial q^j} \Gamma_r(x, q) \right]_{q=0} & = -\delta_{ij} 2G^2 N_c N_f \frac{1}{\pi^2} \int_0^\Lambda dp \frac{1}{E_p} \left[1 - \frac{1}{e^{\beta(E_p - \mu)} + 1} \right. \\
& \quad \left. - \frac{1}{e^{\beta(E_p + \mu)} + 1} \right],
\end{aligned} \tag{83}$$

Substituting Eqs.(82)(83) into Eqs.(70)(71) and combining Eq.(59), we obtain

$$\begin{aligned}
& \sigma(x) + N_c N_f \frac{1}{\pi^2} \int_0^\Lambda dp p^2 \frac{M}{E_p} \left[1 - \frac{1}{e^{\beta(E_p - \mu)} + 1} - \frac{1}{e^{\beta(E_p + \mu)} + 1} \right] \\
& - \frac{G}{2} N_c N_f \frac{1}{\pi^2} \int_0^\Lambda dp \frac{1}{E_p} \left[1 - \frac{1}{e^{\beta(E_p - \mu)} + 1} - \frac{1}{e^{\beta(E_p + \mu)} + 1} \right] \nabla^2 \sigma(x) \\
& = 0,
\end{aligned} \tag{84}$$

which is the Ginzburg-Landau equation for the inhomogeneous quark matter in the NJL model.

We can also consider the time variation of the quark matter near the critical point by the same method that we applied to the space coordinates, which gives us the time-dependent Ginzburg-Landau equation. Without influencing the final result, we can take the limit $\vec{q} \rightarrow 0$ and Eq.(78) is simplified to the following form,

$$\begin{aligned}
\Gamma_r(x, q)_{\vec{q}=0} & = -2G \\
& + 4G^2 N_c N_f \int \frac{d^3 p}{(2\pi)^3} \left(\tanh \left[\frac{\beta}{2} (E_p - \mu) \right] + \tanh \left[\frac{\beta}{2} (E_p + \mu) \right] \right) \\
& \times \frac{1}{E_p} \left(\frac{2M^2 + E_p q_0}{(q_0 + E_p)^2 - E_p^2 + i\epsilon(q_0 + E_p)} \right)
\end{aligned}$$

$$+ \frac{2M^2 - E_p q_0}{(q_0 - E_p)^2 - E_p^2 + i\epsilon(q_0 - E_p)} \Big). \quad (85)$$

In the same way, we ignore the mass term because the system is near the QCD critical point. By expanding Eq.(85) with respect to q_0 , we obtain

$$\begin{aligned} \Gamma_r(x, q)_{\vec{q}=0} &= -2G \\ &+ 4G^2 N_c N_f \frac{1}{\pi^2} \int_0^\Lambda dp \frac{p^2}{E_p} \left[1 - \frac{1}{e^{\beta(E_p - \mu)} + 1} - \frac{1}{e^{\beta(E_p + \mu)} + 1} \right] \\ &+ q_0^2 G^2 N_c N_f \frac{1}{\pi^2} \int_0^\Lambda dp \frac{p^2}{E_p^3} \left[1 - \frac{1}{e^{\beta(E_p - \mu)} + 1} - \frac{1}{e^{\beta(E_p + \mu)} + 1} \right]. \end{aligned} \quad (86)$$

Differentiating $\Gamma_r(x, q)$ with respect to q_0 , we have

$$\left[\frac{\partial}{\partial q^0} \Gamma_r(x, q) \right]_{q=0} = 0, \quad (87)$$

$$\begin{aligned} \left[\frac{\partial^2}{\partial q^0 \partial q^0} \Gamma_r(x, q) \right]_{q=0} &= 2G^2 N_c N_f \frac{1}{\pi^2} \int_0^\Lambda dp \frac{p^2}{E_p^3} \left[1 - \frac{1}{e^{\beta(E_p - \mu)} + 1} \right. \\ &\quad \left. - \frac{1}{e^{\beta(E_p + \mu)} + 1} \right], \end{aligned} \quad (88)$$

Substituting Eqs.(87)(88) into Eqs.(70) (71) and combining Eq.(84), we obtain the time-dependent Ginzburg-Landau equation:

$$\begin{aligned} &\sigma(x) + N_c N_f \frac{1}{\pi^2} \int_0^\Lambda dp p^2 \frac{M}{E_p} \left[1 - \frac{1}{e^{\beta(E_p - \mu)} + 1} - \frac{1}{e^{\beta(E_p + \mu)} + 1} \right] \\ &- \frac{G}{2} N_c N_f \frac{1}{\pi^2} \int_0^\Lambda dp \frac{1}{E_p} \left[1 - \frac{1}{e^{\beta(E_p - \mu)} + 1} - \frac{1}{e^{\beta(E_p + \mu)} + 1} \right] \nabla^2 \sigma(x) \\ &+ \frac{G}{2} N_c N_f \frac{1}{\pi^2} \int_0^\Lambda dp \frac{p^2}{E_p^3} \left[1 - \frac{1}{e^{\beta(E_p - \mu)} + 1} - \frac{1}{e^{\beta(E_p + \mu)} + 1} \right] \partial_t^2 \sigma(x) \\ &= 0. \end{aligned} \quad (89)$$

Note that in the above time-dependent Ginzburg-Landau equation, the time derivative is second order, meaning that the σ mode is a propagating one, which is different from the diffusive time-dependent Ginzburg-Landau equation for the precursory mode in the superconductor phenomena [51, 52].

4. Summary and Discussions

CTPGF formalism is a powerful tool, because it can deal with not only equilibrium problems but also nonequilibrium ones. When much attention are paid on the critical dynamics of the QCD phase transitions, especially the chiral phase transition, it is quite natural for people to choose the CTPGF formalism. In this work, the CTPGF formalism is applied in the NJL model. First of all, we simply review the CTPGF formalism, mainly on the generating functional of the CTPGFs. Then we use this formalism to obtain the well-known gap equation for the quark condensate in stationary homogeneous system. We have also used this formalism to obtain the Ginzburg-Landau equation for the chiral order parameter in stationary inhomogeneous system and the time-dependent Ginzburg-Landau equation describing the critical dynamics of the chiral phase transition. In our derived Ginzburg-Landau equation, there is no other parameters except for those in the original NJL model.

In the present paper, we only derive the time-dependent Ginzburg-Landau equation. However, to fully appreciate the critical dynamics of the chiral phase transition, we also need to analyze the equation numerically and make some comparison with the experimental data. Furthermore, the present derivation is only the mean field result and we ignore the effects of the interaction of $\sigma(x)$ mode with $\pi(x)$ meson. Related works are under progress and we will report them elsewhere.

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